Realism in the philosophy of mathematics, we are often told, gives rise to so-called access problems. Similar issues arise for realism in metaethics, or in metanormativity more generally, at least for versions of non-naturalism.

The crucial worry is that, on these views, the relevant facts are sui generis and thus (?) not part of the causal nexus.

These are murky waters, though, since (i) it is never quite clear what ‘realism’ amounts to and (ii) there is much disagreement as to whether access problems have been solved.

Still, one thing is clear: the issues that arise for ethics and mathematics are strikingly similar. Accordingly, we find many of what I will call twin responses:

1. a. Circumvent the problem by positing a quasi-perceptual faculty.
   b. Deny that the putative discourse is onto something.
   c. Insist that the relevant discourse is ‘mere’ pretense.

A glaring exception: expressivism (or: non-cognitivism, emotivism, quasi-realism, etc.).

Although it has received much attention as an alternative to realism in meta-ethics, no-one seems to be defending the analog view in the philosophy of mathematics.

→ Why?

METAETHICAL EXPRESSIVISM: WHAT

Passing familiarity with textbook versions of expressivism might suggest the answer is obvious.

Preliminary characterization: utterances of declarative moral sentences are mere expressions of emotions. (Think: ‘Cannibalism: Boo!’)

Less tendentiously, we could think of it this way:

Preliminary characterization (bis): utterances of declarative moral sentences are expressions of pro- or con- attitudes towards certain actions. (Think: disapproval of cannibalism.)

But then: what is the attitude expressed by a (pure) mathematical statement directed at?

Best to think of metaethical expressivism as a conjunction of two claims:

MENTALISM (about moral language): the meaning of public lan-
guage moral sentences is determined by the role they play as devices for expressing mental states.

NON-REPRESENTATIONALISM (about moral thought): moral thought is non-representational. To believe that cannibalism is wrong does not involve representing that cannibalism is wrong.

The controversial component of metaethical expressivism—what gives rise to its virtues and vices—is the second one.

Now, NON-REPRESENTATIONALISM will not have much content unless we say something about what ‘representation’ means.

On a family of views that became popular in the 1980s, to represent that $p$ is to be in a state that causally co-varies with the state that $p$. But surely, not all varieties of non-naturalism are varieties of expressivism. Better to understand ‘representation’ along different lines. But how?

A way around this: replace NON-REPRESENTATIONALISM with:

A-REPRESENTATIONALISM (about moral thought): Representational relations of any sort between ourselves and moral facts have no role to play in our best theories of moral thought.

This is a much more general understanding of expressivism, which clears out conceptual space for expressivism in a wide variety of areas. In particular, it opens the door to a kind of expressivism about mathematics. More on this below.

Note though: a commitment to A-REPRESENTATIONALISM (about moral thought) gives rise to the question:

How else should we go about giving a theory about what it is to think that cannibalism is wrong?

This is where most of the action takes place.

For example, according to Gibbard, to think that cannibalism is wrong is to be in a complex dispositional state that involves relations between patterns of behavior and certain emotions.

And whatever the answer, that answer had better satisfy a number of desiderata, e.g.

(2) a. It should explain the role that the relevant states play in our mental economy.

b. It should explain why the relevant states of mind bear logical relations to one another. In particular, it should explain why the relevant family of states forms a Boolean algebra.

c. It should be amenable to a story about why we engage in the relevant discourse.
EXPRESSIVISM ABOUT MATHEMATICS: WHY NOT?

There is conceptual room for a form of expressivism about mathematics. Is it a non-starter?

To start, consider three putative reasons for thinking that it is.

(3) a. It involves a revision of mathematics.
   b. It is incompatible with the objectivity of mathematics.
   c. Any plausible account of mathematical thought must appeal to representational relations.

On the first: to the extent that mathematical practice makes appeal to a notion of truth, it can be understood in purely deflationary terms. More importantly, the stated aim of A-REPRESENTATIONALISM is to give an account of mathematical practice as it is without making appeal to substantive word-world relations (or thought-world relations).

On the second: this is a tricky issue, in part because it is not clear what 'objectivity' amounts to. But to the extent that we would be satisfied with some form of 'intersubjectivity', we may be able to account for objectivity in that sense even if we opt for A-REPRESENTATIONALISM.

Finally, on the third: best to take it as a challenge. Is there a way of thinking about mathematical thought that does not appeal to representational relations between our mental states and mathematical facts?

A PROGRESS REPORT

The question is essentially one about mathematical concepts. How should we think of mathematical concepts if not on representational terms?

Start out by noting that there hasn't been yet any illuminating account of mathematical thought in representational terms.

Indeed, at least on modest naturalistic assumptions about ourselves, we haven't a clue as to how to give an account of what representation involves.

What's more, we have a number of relatively successful historical explanations of the emergence of certain bits of mathematics which do not appeal to cognitive relations between ourselves and mathematical objects.

It certainly seems plausible that if you can give an adequate account of the emergence of a range of conceptual tools without appealing to thought-world relations, you can give an account of the relevant concepts that is compatible A-REPRESENTATIONALISM. (This is not to say, of course, that such an account is in the offing.)

That might help shift the burden of proof a little.

Still, the question remains wide open. What is it for someone to have a particular mathematical concept?

I'm tempted by what is essentially a form of inferentialism. First, think of mathematical concepts as being variable-like. Mathematical concepts, on this picture, are like schematic letters: their functional role is fixed by the...
inferential relations it bears to other mathematical concepts, and by the way they can be 'linked' to empirical concepts.

To accept a mathematical theory is just to adopt one such system of concepts.

I suspect that the claim that the relevant concepts can be accounted for without appealing to representational relations may sound attractive to many. What I want to emphasize here is that this is but a short step from a form of expressivism about mathematics.

What, after all, would the alternative be? That when accepting a mathematical theory we are representing facts about our concepts?

**Precedents & Re-Orientation**

The idea that mathematical discourse is not 'fact-stating' was not unheard of during the first half of the 20th century.

But historical precedent goes quite a bit further. Famously, Frege's understanding of formalism (on which mathematics is about physical signs) does not fit the pattern. But some of the things Hilbert said do smack of a-representationalism:

To make it a universal requirement that each individual formula ...be interpretable by itself is by no means reasonable; on the contrary, a theory by its very nature is such that we do not need to fall back upon intuition or meaning in the midst of some argument. What the physicist demands precisely of a theory is that particular propositions be derived from laws of nature or hypotheses solely by inferences, hence on the basis of a pure formula game, without extraneous considerations being adduced. Only certain combinations and consequences of physical laws can be checked by experiment—just as in my proof theory only the real propositions are directly capable of verification.

And on some interpretations, Bishop Berkeley may well have held a similar view.

But let us set historical pedigree aside. In closing, I want to outline what I take to be the main benefits of a successful a-representationalist account of mathematical thought.

- First, it shifts the emphasis away from questions about the 'nature' of mathematical objects and questions about 'access' to questions about **conceptual engineering**.
- Second, it explains the alleged incoherence of an 'Evil Demon' scenario in the case of mathematics.
- Finally, it avoids the trappings of the Model-in-the-Sky picture sometimes associated with Platonism.

These may not be reasons for believing in a-representationalism about mathematics. But they sure seem to be reasons for giving it some attention.
REFERENCES


