MATHEMATICS WITHOUT REPRESENTATION?

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Realism in the philosophy of mathematics, we are often told, gives rise to so-called *access problems*. Similar issues arise for realism in metaethics, or in metanormativity more generally, at least for versions of *non-naturalism*.

The usual suspects include Benacerraf 1973, Field 1989, Hodes 1984.

Here I'm thinking of Parfit 2011, Dworkin 2011, Scanlon 1998, Moore 1903.

> The crucial worry is that, on these views, the relevant facts are *sui generis* and thus (?) not part of the causal nexus.

These are murky waters, though, since (i) it is never quite clear what 'realism' amounts to and (ii) there is much disagreement as to whether access problems have been solved.

Some even insist there's no real problem to be solved—e.g. J. P. Burgess & Rosen 1997.

Still, one thing is clear: the issues that arise for ethics and mathematics are strikingly similar. Accordingly, we find many of what I will call *twin responses*:

Cf. 'companions in guilt' arguments. For relevant discussion and references, see e.g. Clarke-Doane 2012.

(1) a. Circumvent the problem by positing a quasi-perceptual faculty. (E.g. Gödel 1964, Ross 1927, Enoch 2011.)
b. Deny that the putative discourse is onto something. (E.g. Mackie 1977, Field 1980.)
c. Insist that the relevant discourse is 'mere' pretense. (E.g. Kalderon 2005, Yablo 2001.)

A glaring exception: *expressivism* (or: *non-cognitivism, emotivism, quasi-realism,* etc.).

Gibbard 1990, 2003, Blackburn 1998.

> Although it has received much attention as an alternative to realism in meta-ethics, no-one seems to be defending the analog view in the philosophy of mathematics.
>
> $\rightsquigarrow$ Why?

The literature here is vast. For critical discussion see e.g. Schroeder 2008, 2010b, van Roojen 1996.

METAETHICAL EXPRESSIVISM: WHAT

Passing familiarity with textbook versions of expressivism might suggest the answer is obvious.

> *Preliminary characterization*: utterances of declarative moral sentences are mere expressions of emotions. (Think: 'Cannibalism: Boo!')

Cf. Barnes 1934.

Less tendentiously, we could think of it this way:

> *Preliminary characterization (bis)*: utterances of declarative moral sentences are expressions of pro- or con- attitudes towards certain actions. (Think: disapproval of cannibalism.)

Cf. Ayer 1936. There's also the possibility of thinking of them as imperatives—e.g. Carnap 1937, Hare 1952.

But then: what is the attitude expressed by a (pure) mathematical statement directed *at*?

Best to think of metaethical expressivism as a conjunction of two claims:

> MENTALISM (about moral language): the meaning of public lan-

guage moral sentences is determined by the role they play as devices for expressing mental states.

NON-REPRESENTATIONALISM (about moral thought): moral thought is *non-representational*. To believe that cannibalism is wrong does not involve representing that cannibalism is wrong.

It also does not involve representing that one disapproves of cannibalism, say.

The controversial component of metaethical expressivism—what gives rise to its virtues and vices—is the second one.

MENTALISM has acquired the status of orthodoxy since at least Grice 1957, 1969. See also Davis 2002, Lewis 1975, Stalnaker 1984. Notable exceptions include Davidson 1974, Dummett 1991.

Now, NON-REPRESENTATIONALISM will not have much content unless we say something about what 'representation' means.

On a family of views that became popular in the 1980s, to *represent* that p is to be in a state that *causally co-varies* with the state that p. But surely, not all varieties of non-naturalism are varieties of expressivism. Better to understand 'representation' along different lines. But how?

Cf. Stalnaker 1984, Dretske 1981, Fodor 1987, Millikan 1984, *inter alia*. This is also the notion of representation operative in much of cognitive science. Cf. Gallistel & King 2009, p. 55f.

A way around this: replace NON-REPRESENTATIONALISM with:

A-REPRESENTATIONALISM (about moral thought): Representational relations of any sort between ourselves and moral facts have no role to play in our best theories of moral thought.

This may need to be qualified: perhaps the view is that no 'substantial' representational relations have a role to play in our best theories; or that no representational relations have any *explanatory* role to play in our best theories. Arguably, neither of these views conflicts with modern semantic theory—or so I argue in my 2014a.

This is a much more general understanding of expressivism, which clears out conceptual space for expressivism in a wide variety of areas. In particular, it opens the door to a kind of *expressivism about mathematics*. More on this below.

Cf. Yalcin 2011, Price 1983, Blackburn 1993, Schroeder 2010a, Brandom 1994. Indeed, this makes 'global expressivism' (cf. Price 2013) a theoretical possibility.

Note though: a commitment to A-REPRESENTATIONALISM (about moral thought) gives rise to the question:

How else should we go about giving a theory about what it is to think that cannibalism is wrong?

This is where most of the action takes place.

For example, according to Gibbard, to think that cannibalism is wrong is to be in a complex dispositional state that involves relations between patterns of behavior and certain emotions.

Gibbard is thus very much part of the Sentimentalist tradition exemplified by David Hume and Adam Smith.

And whatever the answer, that answer had better satisfy a number of desiderata, e.g.

(2) a. It should explain the role that the relevant states play in our mental economy.
b. It should explain why the relevant states of mind bear logical relations to one another. In particular, it should explain why the relevant family of states forms a Boolean algebra.
c. It should be amenable to a story about why we engage in the relevant discourse.

There is a venerable tradition that arrives at A-REPRESENTATIONALISM through a different route, for which the project of accounting for our linguistic practices is primary—e.g. Brandom 1994, Williams 1999, Price 2004. Here, though, I take MENTALISM for granted.

EXPRESSIVISM ABOUT MATHEMATICS: WHY NOT?

There is conceptual room for a form of expressivism about mathematics. Is it a non-starter?

To start, consider three putative reasons for thinking that it is.

(3) a. It involves a revision of mathematics.
b. It is incompatible with the objectivity of mathematics.
c. Any plausible account of mathematical thought must appeal to representational relations.

On the *first*: to the extent that mathematical practice makes appeal to a notion of truth, it can be understood in purely deflationary terms. More importantly, the stated aim of A-REPRESENTATIONALISM is to give an account of mathematical practice *as it is* without making appeal to substantive word-world relations (or thought-world relations).

Cf. Tait 1986.

On the *second*: this is a tricky issue, in part because it is not clear what 'objectivity' amounts to. But to the extent that we would be satisfied with some form of 'intersubjectivity', we may be able to account for objectivity *in that sense* even if we opt for A-REPRESENTATIONALISM.

Cf. Rosen 1994. Also, for the various senses of the term as used in the history of science, see Daston & Gallison 2007.

cf. Feferman 2009: " The objectivity of mathematics lies in its stability and coherence under repeated communication, critical scrutiny and expansion by many individuals often working independently of each other. [...] The objectivity of mathematics is a special case of intersubjective objectivity that is ubiquitous in social reality."

Finally, on the *third*: best to take it as a challenge. Is there a way of thinking about mathematical thought that does not appeal to representational relations between our mental states and mathematical facts?

A PROGRESS REPORT

The question is essentially one about mathematical *concepts*. How should we think of mathematical concepts if not on representational terms?

A hint: "mathematics [is] a linguistic artifact, something we have designed, and continue to design, to help us get by in the world. As a community of practitioners, we choose to do mathematics the way we do, and when we do mathematics in the usual ways, we are bound by these choices." Avigad to appear, p. 4.

Start out by noting that there hasn't been yet any *illuminating* account of mathematical thought in representational terms.

> Indeed, at least on modest naturalistic assumptions about ourselves, we haven't a clue as to how to give an account of what representation involves.

Cf. Hodes 1984.

What's more, we have a number of relatively successful historical explanations of the emergence of certain bits of mathematics which do not appeal to cognitive relations between ourselves and mathematical objects.

Cf. the discussion of the discovery of complex numbers and the discovery of groups in Kitcher 2012. Cf. also Maddy 2011, ch. 2.

> It certainly *seems* plausible that if you can give an adequate account of the emergence of a range of conceptual tools without appealing to thought-world relations, you can give an account of the relevant concepts that is compatible A-REPRESENTATIONALISM. (This is not to say, of course, that such an account is in the offing.)

That might help shift the burden of proof a little.

Still, the question remains wide open. What is it for someone to have a particular mathematical concept?

I'm tempted by what is essentially a form of *inferentialism*. First, think of mathematical concepts as being variable-like. Mathematical concepts, on this picture, are like schematic letters: their functional role is fixed by the

To some extent, this is a view like the 'Aristotelianism' outlined in Pettigrew 2008. The main, and to my mind crucial difference, is that unlike Pettigrew I do not resort to universal closure in order to get a descriptive content—Pettigrew's stated goal is to respond to the objection that structuralism requires a revision of our understanding of everyday mathematical language.

inferential relations it bears to other mathematical concepts, and by the way they can be 'linked' to empirical concepts.

> To accept a mathematical theory is just to adopt one such system of concepts.

For a more direct attempt at getting here, see my 2014b.

I suspect that the claim that the relevant concepts can be accounted for without appealing to representational relations may sound attractive to many. What I want to emphasize here is that this is but a short step from a form of *expressivism* about mathematics.

> What, after all, would the alternative be? That when accepting a mathematical theory we are representing facts about *our concepts*?

Something like this seems to be the view that Martin 2005 attributes to Gödel.

PRECEDENTS & RE-ORIENTATION

The idea that mathematical discourse is not 'fact-stating' was not unheard of during the first half of the 20th century.

E.g. Ayer 1936, Gasking 1940 and, notoriously, Wittgenstein [1953] 2009, 1978. A more recent example, plausibly, is Kitcher 2012, p. 187ff: "*the language-games* [mathematicians] *play* just aren't in the description business."

But historical precedent goes quite a bit further. Famously, Frege's understanding of formalism (on which mathematics is about physical signs) does not fit the pattern. But some of the things Hilbert said do smack of A-REPRESENTATIONALISM:

Here again, cf. Detlefsen 2005, who reads Hilbert as subscribing to a Berkeleyan conception of reasoning, and not to the version of formalism ('empiricist formalism') criticized in Frege 1891.

> To make it a universal requirement that each individual formula …be interpretable by itself is by no means reasonable; on the contrary, a theory by its very nature is such that we do not need to fall back upon intuition or meaning in the midst of some argument. What the physicist demands precisely of a theory is that particular propositions be derived from laws of nature or hypotheses solely by inferences, hence on the basis of a pure formula game, without extraneous considerations being adduced. Only certain combinations and consequences of physical laws can be checked by experiment—just as in my proof theory only the real propositions are directly capable of verification.

Hilbert 1928, p. 475, as cited in Detlefsen 2005, p. 297f.

And on some interpretations, Bishop Berkeley may well have held a similar view.

Cf. Detlefsen 2005, p. 268: "The cognitive significance of language was therefore not, in Berkeley's view, exclusively a result of its semantic usage. It could, in addition, have a kind of logistic use whose aim was to assist the mind in reasoning, judging, extending, recording, and communicating knowledge even when it was not used to express ideas."

But let us set historical pedigree aside. In closing, I want to outline what I take to be the main benefits of a successful a-representationalist account of mathematical thought.

- First, it shifts the emphasis away from questions about the 'nature' of mathematical objects and questions about 'access' to questions about *conceptual engineering*.
- Second, it explains the alleged incoherence of an 'Evil Demon' scenario in the case of mathematics.
- Finally, it avoids the trappings of the Model-in-the-Sky picture sometimes associated with Platonism.

Or 'conceptual ethics', to borrow the terminology of A. Burgess & Plunkett 2013. Cf. Avigad to appear.

Cf. Maddy 2011, ch. 3.

Cf. Tait 1986, 2001.

These may not be reasons for believing in A-REPRESENTATIONALISM about mathematics. But they sure seem to be reasons for giving it some attention.

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